

$$61. y = -2 \cos(2x + \frac{\pi}{2}) - 1$$

$$y = -2 \cos[2(x + \frac{\pi}{4})] - 1$$

Amplitude: 2

$$\text{period: } \frac{2\pi}{|b|} = \frac{2\pi}{2} = \pi$$

Phase shift: $-\pi/4$ or left $\pi/4$

Vertical shift: down 1 or -1

5- Key points

$$y = \cos x$$

$$(0, 1)$$

$$(\pi/2, 0)$$

$$(\pi, -1)$$

$$(3\pi/2, 0)$$

$$(2\pi, 1)$$

5-key points for

$$y = -2 \cos[2(x + \pi/4)] - 1$$

$$(\frac{x}{2} - \frac{\pi}{4}, -2y - 1)$$

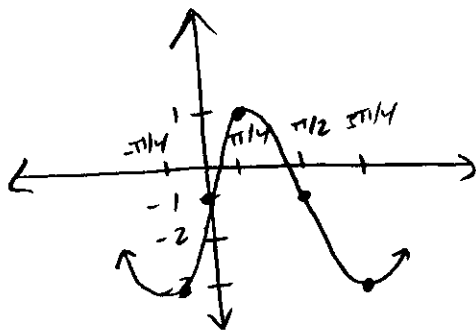
$$(\frac{0}{2} - \frac{\pi}{4}, -2(1) - 1) = (-\frac{\pi}{4}, -3)$$

$$(\frac{\pi}{2} \cdot \frac{1}{2} - \frac{\pi}{4}, -2(0) - 1) = (0, -1)$$

$$(\pi \cdot \frac{1}{2} - \frac{\pi}{4}, -2(-1) - 1) = (\frac{\pi}{4}, 1)$$

$$(\frac{3\pi}{2} \cdot \frac{1}{2} - \frac{\pi}{4}, -2(0) - 1) = (\frac{\pi}{2}, -1)$$

$$(2\pi \cdot \frac{1}{2} - \frac{\pi}{4}, -2(1) - 1) = (\frac{3\pi}{4}, -3)$$



62. $y = 3\sin[2(x - \pi/6)] + 1$

Amplitude: 3

Period: $\frac{2\pi}{2} = \pi$

phase shift: right $\pi/6$

vertical shift: up 1

$y = \sin x$

5 key points

$(0, 0)$

$(\pi/2, 1)$

$(\pi, 0)$

$(3\pi/2, -1)$

$(2\pi, 0)$

$y = 3\sin[2(x - \pi/6)] + 1$

$(\frac{x}{2} + \frac{\pi}{6}, 3y+1) \rightarrow (\frac{\pi}{6}, 1)$

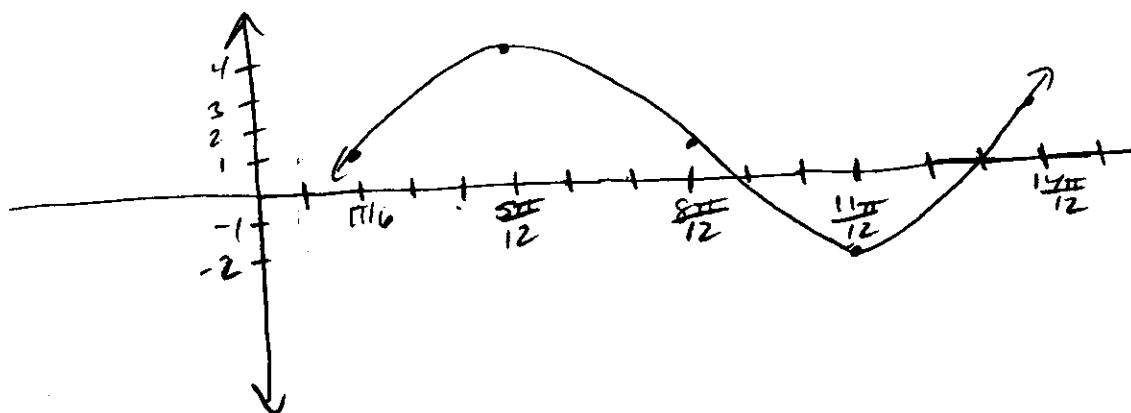
$(\frac{\pi}{2} + \frac{\pi}{6}, 1) \rightarrow (\frac{5\pi}{12}, 4)$

$(\frac{\pi}{2} \cdot \frac{1}{2} + \frac{\pi}{6}, 3(1)+1) \rightarrow (\frac{5\pi}{12}, 4)$

$(\frac{\pi}{4} + \frac{\pi}{6}, 3(0)+1) \rightarrow (\frac{5\pi}{12}, 1)$

$(\frac{3\pi}{2} \cdot \frac{1}{2} + \frac{\pi}{6}, 3(-1)+1) \rightarrow (\frac{11\pi}{12}, -2)$

$(2\pi \cdot \frac{1}{2} + \frac{\pi}{6}, 3(0)+1) \rightarrow (\frac{13\pi}{12}, 1)$



63. $y = \csc(x + \frac{\pi}{2}) - 1$

Period: 2π

Phase shift: left $\pi/2$ or $-\pi/2$

Vertical shift: down 1 or -1

Asymptotes: $\frac{\pi}{2} + \pi k$

5-key points.

$y = \csc x \rightarrow \frac{1}{\sin x}$

$(0, \text{und.})$

$(\frac{\pi}{2}, 1)$

$(\pi, \text{und.})$

$(\frac{3\pi}{2}, -1)$

$(2\pi, \text{und.})$

5-key points

$y = \csc(x + \frac{\pi}{2}) - 1$

$(x - \frac{\pi}{2}, y - 1)$

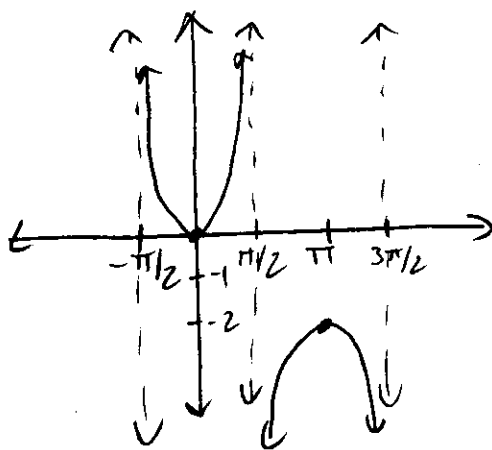
$(-\pi/2, \text{und.})$

$(0, 0)$

$(\pi/2, \text{und.})$

$(\pi, -2)$

$(\frac{3\pi}{2}, \text{und.})$



64. $y = -4\sec 2(x + \frac{\pi}{3})$

5-key points
 $y = \sec x = \frac{1}{\cos x}$

$(0, 1)$

$(\pi/2, \text{und.})$

$(\pi, -1)$

$(\frac{3\pi}{2}, \text{und.})$

$(2\pi, 1)$

5-key points

$y = -4\sec 2(x + \frac{\pi}{3})$

$(\frac{\pi}{3}, -4)$

$(-\frac{\pi}{3}, -4) \rightarrow (-\frac{\pi}{3}, -4)$

$(\frac{\pi}{2} - \frac{\pi}{3}, \text{und.}) \rightarrow (-\frac{\pi}{6}, \text{und.})$

$(\pi - \frac{\pi}{3}, 4) \rightarrow (\frac{2\pi}{3}, 4)$

$(\frac{3\pi}{2} - \frac{\pi}{3}, \text{und.}) \rightarrow (\frac{5\pi}{2}, \text{und.})$

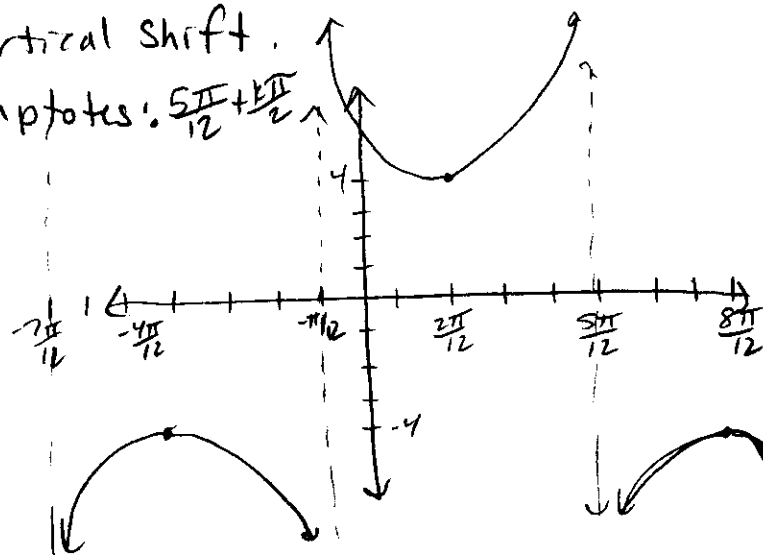
$(2\pi - \frac{\pi}{3}, -4) \rightarrow (\frac{5\pi}{2}, -4)$

Period: π

Phase shift: left $\pi/3$ or $-\pi/3$

No vertical shift.

Asymptotes: $\frac{5\pi}{12} + \frac{k\pi}{2}$



65. $y = \tan\left(\frac{\pi}{4}x - \frac{3\pi}{4}\right)$
 $y = \tan\left[\frac{\pi}{4}(x-3)\right]$
 period: $\frac{\pi}{\frac{\pi}{4}} = \pi \cdot \frac{4}{\pi} = 4$

Phase shift: right 3

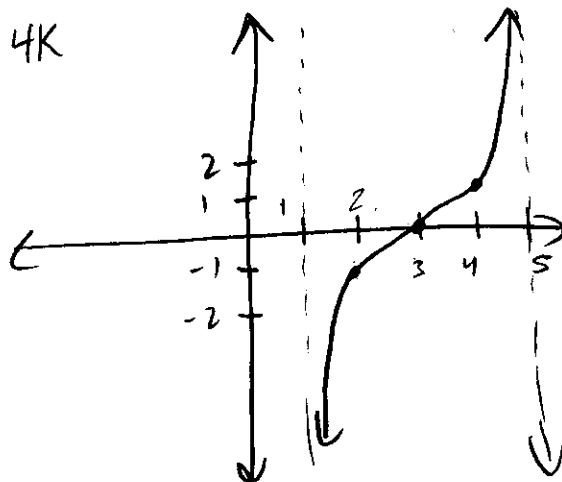
no vert. shift.

Asymptotes: $1 + 4K$

5 key points:
 $y = \tan x = \frac{\sin x}{\cos x}$

$(-\frac{\pi}{2}, \text{und})$
 $(-\frac{\pi}{4}, -1)$
 $(0, 0)$
 $(\frac{\pi}{4}, 1)$
 $(\frac{\pi}{2}, \text{und})$

$y = \tan \frac{\pi}{4}(x-3)$
 $(x \cdot \frac{4}{\pi} + 3, y)$
 $(-\frac{\pi}{2} \cdot \frac{4}{\pi} + 3, \text{und}) \rightarrow (1, \text{und})$
 $(-\frac{\pi}{4} \cdot \frac{4}{\pi} + 3, -1) \rightarrow (2, -1)$
 $(0 \cdot \frac{4}{\pi} + 3, 0) \rightarrow (3, 0)$
 $(\frac{\pi}{4} \cdot \frac{4}{\pi} + 3, 1) \rightarrow (4, 1)$
 $(\frac{\pi}{2} \cdot \frac{4}{\pi} + 3, \text{und}) \rightarrow (5, \text{und})$

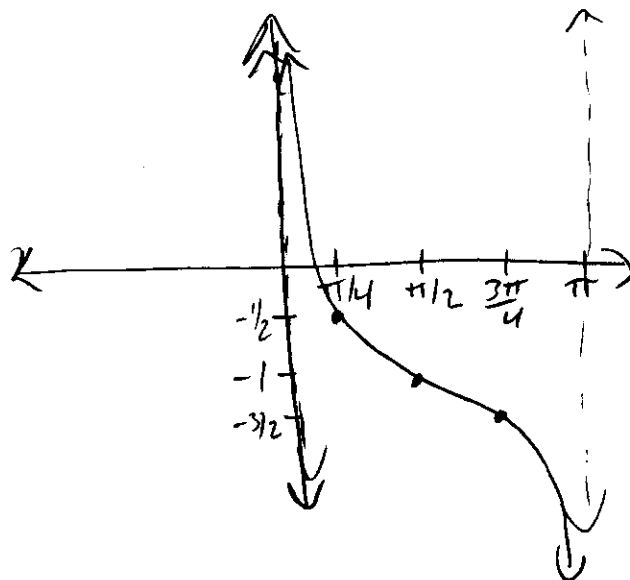


66. $y = \frac{1}{2} \cot x - 1$
 period: π
 phase shift: none
 vert. shift: down 1
 Asymptotes:

5 key points
 $y = \cot x = \frac{\cos x}{\sin x}$

$(0, \text{und})$
 $(\pi/4, 1)$
 $(\pi/2, 0)$
 $(3\pi/4, -1)$
 (π, und)

5 key points
 $y = \frac{1}{2} \cot x - 1$
 $x, \frac{1}{2}y - 1$
 $(0, \text{und})$
 $(\pi/4, -1/2)$
 $(\pi/2, -1)$
 $(3\pi/4, -3/2)$
 (π, und)



67. $y = -(3 \sin(x + \frac{\pi}{4}) + 5)$

$$y = -3 \sin(x + \pi/4) - 5$$

68. use $y = a \sin[b(x-c)] + d$

$a = 2$, reflection across x-axis ($-a$),
Shift down 1,

So,

$$y = -2 \sin(x) - 1$$

69. $E = 5.2 \cos(20\pi t)$

$$\text{frequency} = \frac{|b|}{2\pi} \quad b = 20\pi$$

$$\frac{20\pi}{2\pi} = \boxed{10} \text{ cycles per second.}$$